UNIT -2
"COLUMN & STURT"

STURT-

"A member which is carrying a compressive load, it may be Horizontal, vertical, inclined is called sturt".

COLUMN-

"In building structures a vertical member which is carrying compressive load is called column".

General classification of column-

It is classified in 3 types-

(i) Short Column
(ii) Long Column
(iii) Medium Column

(i) Short Column- If the slendrness ratio of the column is less than 32 is called short column.

$\lambda < 32$

(ii) Long Column - If the slenderness ratio of the column exceed by 120 is called Long Column.

$\lambda > 120$

(iii) Medium Column- If the Slenderness ratio of a cloumn lies b/w 32 to 120 is called medium column.

$32 \le \lambda \le 120$

Slenderness ratio $(\lambda) \rightarrow \dfrac{\text{effective length}}{\text{min. radivs of gyration}}$

$$\lambda = \frac{\text{Leff.}}{\gamma_{min.}}$$

Note

(i) Short column are fail only in crusing.
(ii) Long column are fail only buckling or bending
(iii) Medium column are fail may be in buckling & may be incrusing

Calculation of the load carrying capacity of a column.

Column Load carrying capacity calculate करने के लिए निम्नलिखित theory दी गई-

(i) Euler Column theory
(ii) Rankine Column theory

(i) Euler Column theory- Euler ने अपनी theory केवल Long column का Load calculate करने के लिए दी थी। और Euler कि theory bending stress पर आधारित होती अर्थात् Euler कि Column theory में bending stress को consider किया गया, जबकि direct stress को negligible माना गया।

Assumptions in Ehler column theory:-

1. Initially the column is perfectly straight and load applied on column should be Axially.
2. The crossection of column is uniform throughout its length
3. The column is perfectly elastic, Homogneous and isotropic and obey to Hook's law.
4. The length of column is very large compare to its width (crossection dimension)
5. The failure of column due to buckling alone.
6. The shortening o the column due to direct compression is negligible.

Load carrying capacity by Euler Theory:

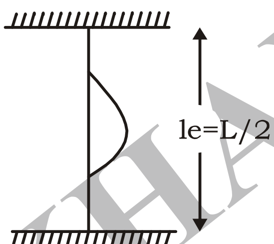
$$P_E = \frac{\pi^2 EI_{min}}{le^2}$$

Load carrying capacity are depended in Euler column theory

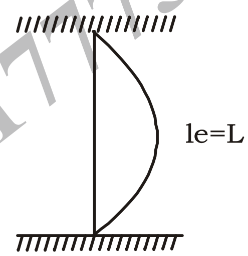
(i) Types of material
(ii) Crossection dimension or lateral dimension
(iii) END restrained condition

Effective length for different END condition:

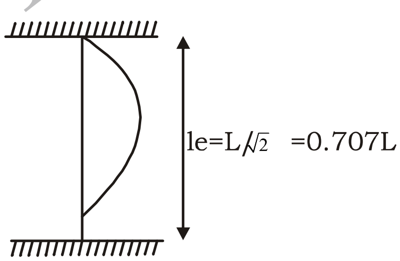
(1) Both end fixed



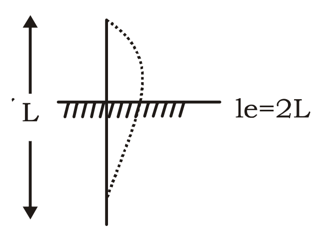
(3) Both end hinged



(2) One end fixed other end hinged



(4) One end fixed other free



S.No.	End Condition	Effective Lenght	Load carrying capacity in term of unsupported Length	Contant (K)
1.	Both end Fixed	leff= $\frac{L}{2}$	$P_E = \frac{\pi^2 EI_{min}}{\left(\frac{L}{2}\right)^2} = \frac{4\pi^2 EI_{min}}{L^2}$	4.0

2. One end fixed other end hinged — Leff= $\frac{L}{\sqrt{2}}$ — $P_E = \frac{\pi^2 EI_{min}}{\left(\frac{L}{\sqrt{2}}\right)^2} = \frac{2\pi^2 EI_{min}}{L^2}$ — 2.0

3. Both end hinged /pinned — Leff=L — $P_E = \frac{\pi^2 EI_{min}}{(L)^2} = \frac{\pi^2 EI_{min}}{L^2}$ — 1.0

4. One end fixed other end free — Leff=2L — $P_E = \frac{\pi^2 EI_{min}}{(2L)^2} = \frac{\pi^2 EI_{min}}{4L^2}$ — $\frac{1}{4}$

NOTE:-

Both end fix के case में column कि Load carrying capacity Maximum होती है।

One end fix other end free case में column Load carrying capacity is minimum.

Slendeness Ratio :- (λ)

$$\lambda = \frac{Left}{\gamma \min.}$$

minimum radius of gyration $\gamma \min. = \sqrt{\frac{I}{A}}$

For circular section $\gamma \min. = \sqrt{\frac{\frac{\pi}{64} \times d^4}{\frac{\pi}{4} \times d^2}} = \sqrt{\frac{d^2}{16}} = \frac{d}{4}$

For square section $\gamma \min. = \sqrt{\frac{I}{A}}$

$$= \sqrt{\frac{a^4/12}{a^2}} = \sqrt{\frac{a^2}{12}} = \frac{a^2}{\sqrt{4 \times 3}}$$

$\gamma \min. = \frac{a}{2\sqrt{3}}$ where 'a' is the side of square section

For Rectangular section $r_{xx} = \sqrt{\frac{I_{x-x}}{A}}$

$$r_{x-x} = \sqrt{\frac{bd^3/12}{bd}} = \sqrt{\frac{d^2}{12}}$$

$$r_{x-x} = \frac{d}{2\sqrt{3}}$$

$$r_{y-y} = \sqrt{\frac{I_{y-y}}{A}}$$

$$= \sqrt{\frac{db^3/12}{bd}} = \sqrt{\frac{b^2}{12}}$$

$$r_{y-y} = \frac{b}{2\sqrt{3}}$$

Slenderness ratio $\lambda = \frac{left}{r_{\min}}$

Specially for circular section

$$r_{\min} = \frac{d}{4}$$

$$\lambda = \frac{left.}{d/4}$$

$$\lambda = \frac{4left.}{d}$$

* Slenderness Ratio $\lambda = \frac{left.}{r_{\min}}$

(i) For both end fixed leff= $\frac{L}{2}$

$$\lambda = \frac{L}{2\, r_{\min}}$$

(ii) For one fixed other hinged leff= $\frac{L}{\sqrt{2}}$

$$\lambda = \frac{L}{\sqrt{2}\, r_{\min}}$$

(iii) Both end hinged/pinned leff = L

$$\lambda = \frac{L}{r_{\min}}$$

(iv) One end fixed other free leff= 2L

$$\lambda = \frac{2L}{r_{\min}}$$

Limitations of Euler column Theory :-

Euler ने अपना experiment mild steel के column पर किया था।
we know that
1 mpa = 1 N/mm^2

$$P_E = \frac{\Pi^2 E I_{min}}{l_{e^2}}$$

I = Ak^2 $\qquad$ $K = \sqrt{\frac{I}{A}}$

$$P_E = \frac{\Pi^2 E A K^2}{l_{e^2}}$$

$$\frac{P_E}{A} = \frac{\Pi^2 E}{\left(\frac{l_e}{K}\right)^2}$$

$$\frac{P_E}{A} = \sigma, \left(\frac{l_e}{K}\right)^2 = \lambda$$

$$\sigma = \frac{\Pi^2 E}{\lambda^2}$$

$$\lambda = \sqrt{\frac{\Pi^2 E}{\sigma}}$$

Types of steel	Crushing stress
Mild steel	320 mpa
Cast Iron	550 mpa
Wrought Iron	250 mpa
Timber	40 mpa

For mild steel σ = 320 N/mm²

$$\lambda = \sqrt{\frac{\Pi^2 X 200 X 10^3 \ N/mm^2}{320 \ N/mm^2}}$$

λ = 78.53   80

Note-Mild steel पर experiment करने पर पाया गया Slenderness ratio 80 तक के column crushing में fail होते है, जबकि Euler column theory केवल long columns के लिए दी गई थी, इसलिए Euler column theory उन्ही column पर valid होती है जिनका slenderness ratio 80 से ज्यादा होता है।

(ii) **RANKINE COLUMN THEORY**

Rankine column theory, short column or long column सभी के लिए valid होती है इसलिए rankine कि column theory में slenderness ratio कि कोई limit नही होती है।

$$\frac{1}{P_R} = \frac{1}{P_E} + \frac{1}{P_c}$$

$$\frac{1}{P_R} = \frac{P_c + P_E}{P_E \ X \ P_c}$$

$$P_R = \frac{P_E \ X \ P_C}{P_C + P_E}$$

where,

P_R = load carrying capacity by Rankine

P_E = Euler, load carrying capacity of column

P_C = Crushing load carrrying capacity (P_C = σ_C .A)

$$P_R = \frac{P_E \ X \ P_C}{P_E + P_C}$$

$$P_R = \frac{P_E \ P_C}{P_E \left[1 + \frac{P_c}{P_E}\right]}$$

$$P_R = \frac{\sigma_c . A}{1 + \frac{\sigma_c A}{\frac{\Pi^2 EI}{le^2}}}$$

$$P_R = \frac{\sigma_c . A}{1 + \frac{\sigma_c A}{\frac{\Pi^2 E \ A \ K^2}{le^2}}} = \frac{\sigma_c . A}{1 + \frac{\sigma_c \ . le^2}{\Pi^2 E K^2}}$$

$\frac{\sigma_c}{\Pi^2 E}$ = **a** $\rightarrow$ **Rankine constant**

$$P_R = \frac{\sigma_c . A}{1 + a\left(\frac{le}{K}\right)^2}$$

Q. Find out the Rankine Load carrying capacity. If Euler Load carrying capacity of a column or crushing Load capacity of column is 1500 & 800 KN.

→ Euler load carrying capacity = 1500 KN

Crushing capacity = 800 KN

Rankine Load carrying capacity = ?

$$P_R = \frac{P_E \, X \, P_C}{P_E + P_C}$$

$$= \frac{1500 \, X \, 800}{1500 + 800}$$

= 521.74 KN